HKUST

MATH1013 Calculus IB

Final Examination (White Version) Name:_____

8th Dec 2018 Student ID:_____

12:30-15:30 Lecture Section:_____

Directions:

- This is a closed book examination. **No Calculator is allowed in this examination**.
- DO NOT open the exam until instructed to do so.
- Turn off all phones and pagers, and remove headphones. All electronic devices should be kept in a bag away from your body.
- Write your name, ID number, and Lecture Section in the space provided above, and also in the **Multiple Choice Item Answer Sheet** provided.
- **Make sure you have written and marked your ID number correctly in the I.D. No. Box in the Multiple Choice Item Answer Sheet**.
- DO NOT use any of your own scratch paper, and do not take any scratch paper away from the examination venue.
- When instructed to open the exam, please check that you have **10** pages of questions in addition to the cover page. Two blank pages attached at the end can be torn off and used as scratch paper.
- Answer all questions. Show an appropriate amount of work for each short or long problem. If you do not show enough work, you will get only partial credit.
- You may write on the backside of the pages, but if you use the backside, clearly indicate that you have done so.
- **Cheating is a serious violation of the HKUST Academic Code. Students caught cheating will receive a zero score for the examination, and will also be subjected to further penalties imposed by the University.**

Please read the following statement and sign your signature.

I have neither given nor received any unauthorized aid during this examination. The answers submitted are my own work.

I understand that sanctions will be imposed, if I am found to have violated the University's regulations governing academic integrity.

Student's Signature :

_____

Question No.	Points	Out of
Q. 1-20		60
Q. 21		12
Q. 22		14
Q. 23		14
Total Points		100

Part I: Answer all of the following multiple choice questions.

- Mark your MC answers to the boxes in the **Multiple Choice Item Answer Sheet** provided.
- Mark only one answer for each MC question. Multiple answers entered for each single MC question will result in a 3 point deduction.

Write also your MC question answers in the following boxes for back up use and for paper checking session only. The grading will be based on the answers you mark on the MC item answer sheet.

Question	1	2	3	4	5	6	7	8	9	10
Answer										
Question	11	12	13	14	15	16	17	18	19	20
Answer										

Each of the MC questions is worth 3 points.

1. What is the color version of your exam? No point will be given to you for this question if you do not choose the color version of your exam correctly, or do not mark your ID number correctly in the MC answer form. (Check your ID marking in the MC form right now!!)

   (a) Green (b) Orange (c) White (d) Yellow (e) None of the previous

2. Find the limit: $\lim\limits_{x\to -1^+} \dfrac{\sin[(x+1)(x+2)]}{|x^2-1|}$.

   (a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) 0 (d) -1 (e) 1

3. Find the constant k that makes the following function continuous everywhere.

$$f(x) = \begin{cases} \cos x + \dfrac{e^{kx} - e^{-kx}}{x} & \text{if } x < 0 \\ x^2 - 1 & \text{if } x \geq 0 \end{cases}$$

   (a) -1 (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) 1 (e) does not exist

4. Find the limit

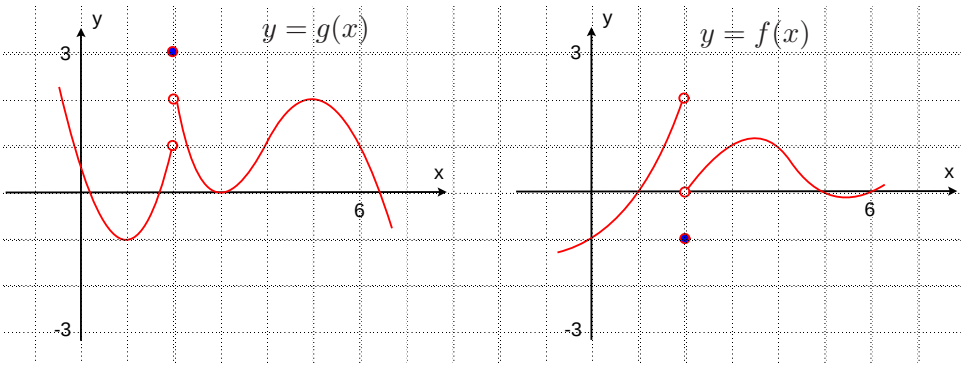
$$\lim_{x\to+\infty} \frac{x-\sqrt{e^x-1}}{x+\sqrt{e^x+2}}\,.$$

(a) $-\infty$ (b) $+\infty$ (c) 0 (d) -1 (e) 1

5. Find the limit: $\lim_{x\to 0}(\cos x)^{\frac{1}{x^2}}$.

(a) $\frac{1}{e}$ (b) $\frac{1}{\sqrt{e}}$ (c) 1 (d) $\sqrt{e}$ (e) e

6. Given the graphs of $y = f(x)$ and $y = g(x)$ as shown below, and let $u(x) = g(x) - f(x)$, find $\lim_{x\to 2^+} f(u(x))$.



(a) does not exist (b) 0 (c) 1 (d) 2 (e) 3

7. Let f be a differentiable function with continuous second derivative f''. Evaluate the limit

$$\lim_{x\to 0} \frac{f(x)+f(-x)-2f(0)}{x^2}\ .$$

(a) $-f''(0)$ (b) $-f'(0)$ (c) 0 (d) $f'(0)$ (e) $f''(0)$

8. Let g be the inverse function of the function f defined by $f(x) = e^x \ln x$ for $x > 0$. Find $g'(0)$.

(a) 1 (b) $\frac{1}{2e}$ (c) $\frac{1}{e}$ (d) e (e) does not exist

9. An elevator moves upward so that at any time $0 \le t \le 5$ (in minutes), its height from the ground is $f(t) = 6t^2 - t^3$ meters. Find the distance travelled by the elevator between the moments when the elevator is at rest.

(a) 0 (b) 6 (c) 12 (d) 16 (e) 32

10. Let C be the curve defined by the equation $y^2+2y = x(x-1)(x+1)$. How many horizontal tangent lines can C have?

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

11. How many real roots does the equation $x^5 - 3x^3 + 1 = 0$ have?

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

12. How many inflection points does the function f have if its derivative function is

$$f'(x) = (x-1)^3(x-3)^2 \ ?$$

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

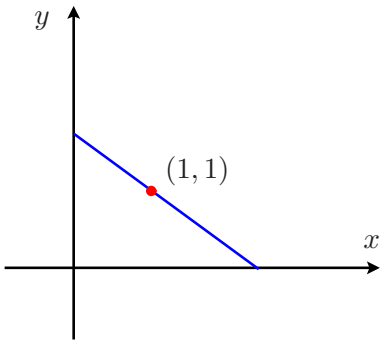
13. Let f be the function defined on $-\infty < x < \infty$ by

$$f(x) = \int_0^x t(e^t - 1)dt$$

At how many points a can $f(a)$ be a local minimum (relative minimum)?

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

14. A line segment L passing through the point $(1, 1)$ has its endpoints lying on the x-axis and y-axis as shown in the figure. What is the shortest possible length of such a line segment?



(a) $\frac{\sqrt{2}}{2}$ (b) $\frac{2\sqrt{2}}{3}$ (c) $\sqrt{2}$ (d) $2\sqrt{2}$ (e) $4\sqrt{2}$

15. If Newton's method is applied to find the critical point of the $f(x) = \sin x - \ln x$, i.e., a point where $f'(x) = 0$, which of the following iteration formulas should be obtained?

(a) $x_{n+1} = x_n - \dfrac{x_n \sin x_n - x_n \ln x_n}{x_n \cos x_n - 1}$

(b) $x_{n+1} = x_n + \dfrac{x_n \sin x_n - \ln x_n}{x_n \cos x_n - 1}$

(c) $x_{n+1} = x_n + \dfrac{x_n^2 \cos x_n - x_n}{x_n^2 \sin x_n - 1}$

(d) $x_{n+1} = x_n - \dfrac{x_n^2 \cos x_n - x_n}{x_n^2 \sin x_n + 1}$

(e) $x_{n+1} = x_n - \dfrac{x_n^2 \sin x_n - 1}{x_n^2 \cos x_n - x_n}$

16. Find the value of the integral $\int_0^{\frac{\pi}{4}} 6\tan^2 x \sec^2 x \, dx$.

(a) 2 (b) 3 (c) 4

(d) 6 (e) 8

17. Find the value of the definite integral $\int_0^{\ln 2} \frac{3e^{-x}}{(1+e^{-x})^2} \, dx$.

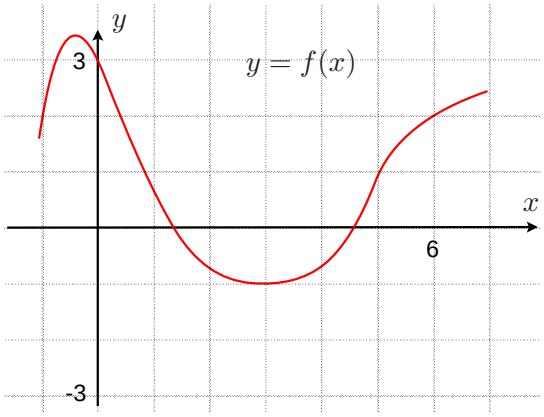
(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) 1 (d) 2 (e) 3

18. Find the limit

$$\lim_{n\to+\infty} \frac{1}{n} \sum_{k=1}^{n} \cos \frac{k\pi}{6n} \ .$$

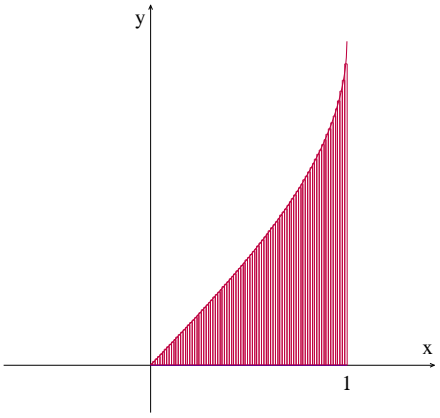
(a) π (b) $\frac{\pi}{2}$ (c) $\frac{2}{3\pi}$ (d) $\frac{2}{\pi}$ (e) $\frac{3}{\pi}$

19. Given the graph of the function $y = f(x)$ as shown below, find $\int_0^5 |f'(x)|(f(x))^2 dx$.



(a) -2 (b) 4 (c) 6 (d) 8 (e) 10

20. Find the area between the graph of $y = 2\arcsin x$ (i.e., $y = 2\sin^{-1} x$) and the x-axis over the interval $[0, 1]$.

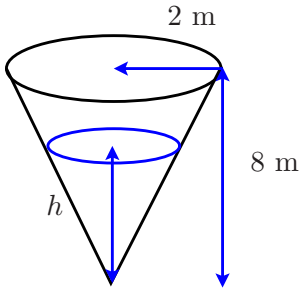


(a) $2\pi - 1$ (b) $2\pi - 2$ (c) $\pi - 1$ (d) $\pi - 2$ (e) $\pi - 3$

Part II: Answer each of the following questions.

21. [*12 pts*] Water is flowing into a tank in the shape of an inverted regular cone at a rate of 2 m^3/min. Suppose the tank has height 8 m and the radius at the top is 2 m, and water in the tank is also leaking out at a constant rate of R m^3/min at the same time.

(a) If the water level is *dropping* at a rate of $\frac{4}{\pi}$ m/min when the height of the water in the tank is 2 m, at what rate R is the water leaking out of the tank? [6 pts]



(b) Suppose instead that water is leaking out of the tank at a changing rate $R(t)$ which is related to the height of water in the tank $h(t)$ at time t by $R = 2 + \pi h^3$. What is the height of the water in the tank at time $t \geq 0$ if the initial height of water in the tank is 6 m? [6 pts]

22. [*14 pts*] Consider the function $f(x) = \dfrac{x}{2} + \dfrac{x}{x^2-2}$ with

x	1	2	3
$f(x)$	$-\frac{1}{2}$	2	$\frac{27}{14}$

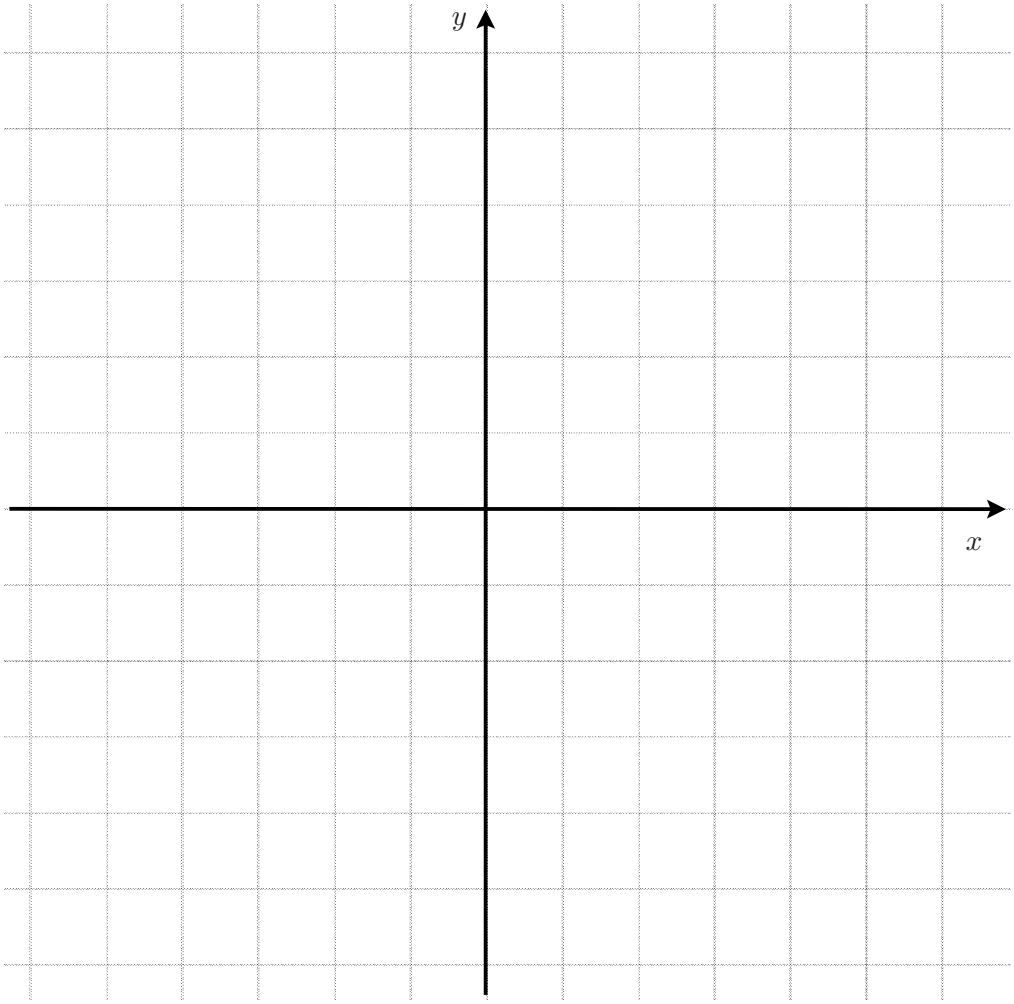
$$f'(x) = \frac{x^4 - 6x^2}{2(x^2-2)^2} \qquad \text{and} \qquad f''(x) = \frac{2x^3 + 12x}{(x^2-2)^3}$$

(a) Find the intervals on which f is increasing. Justify your answer for full credit. [4 pts]

(b) Find the intervals on which f is concave up. Justify your answer for full credit. [3 pts]

(Part (c) to be continued on next page.)

22(c) Using appropriate scales on the axes in the given grid, sketch the graph of f, and indicate all its local maximum or minimum point(s), inflection point(s) and vertical and horizontal asymptotes. [7 pts]



23. [*14 pts*] Let f be the function defined by $f(x) = 8xe^{-2x}$ for $0 \le x \le 3$.

(a) Using the right endpoint Riemann sum on n subintervals of equal length, the integral $\int_0^3 8xe^{-2x}dx$ is given by [2 pts]

$$\int_0^3 8xe^{-2x}dx \;=\; \lim_{n\to\infty} \sum_{k=1}^{n} \boxed{}$$

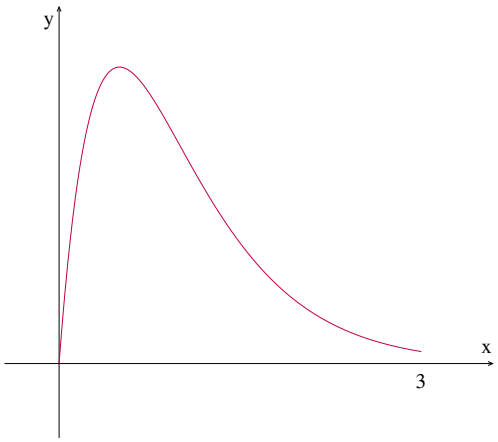
(b) When approximating the definite integral of f over $[0, 3]$ by the *largest possible* Riemann sum on three subintervals of length 1, [8 pts]

$$\int_0^3 f(x)dx \;\approx\; f(c_1) \;+\; f(c_2) \;+\; f(c_3)$$

the sample points to be chosen from the three subintervals are:

$c_1 =$ _____ , $c_2 =$ _____ , $c_3 =$ _____ ,

Justify your answer for full credit.



(c) Is it true that there is a constant c such that $\int_1^3 f(\sin t)dt = 2f(\sin c)$? Justify your answer for full credit. [4 pts]